UDC 531.36+62-50

OPTIMAL CONTROL OF A DYNAMIC SYSTEM WITH RANDOM PARAMETERS UNDER INCOMPLETE INFORMATION

PMM Vol.43, No.4, 1979, pp. 621-628
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(Received April 17, 1976)

An optimal control problem is analyzed for a linear system of ordinary differential equations with random initial data and inhomogeneous terms, when the control performance index is an integral quadratic functional. At each instant the control parameter is chosen on the basis of an observation of the realized values of a specified set of random parameters of the system. Explicit expressions are derived for the optimal control functions and the results obtained are compared with known earlier results.

1. Statement of the problem. We consider the controlled system

$$x^{\cdot} = A(t)x + B(t)u(t, \omega) + f(t, \omega), \quad x(0) = \xi(\omega) \tag{1.1}$$

$$(x \equiv \operatorname{col}\{x_1, \ldots, x_n\}, \quad u(t, \omega) \equiv \operatorname{col}\{u_1(t, \omega), \ldots$$
$$\ldots, u_m(t, \omega)\})$$

where x is the system's state, $u(t, \omega)$ is the vector-valued control function. It is assumed that $A(t)$ and $B(t)$ are matrices of dimensions $n \times n$ and $n \times m$, respectively, with deterministic (i.e., independent of the occurrence of ω) measurable components uniformly bounded on the control interval $[0, T]$. The random initial data vector $\xi(\omega)$ and the random vector-valued function $f(t, \omega)$ of inhomogeneous terms are taken as specified on a complete probability space (Ω, F, P) and satisfy the constraint

$$M\left\{\xi'(\omega)\xi(\omega) + \int_0^T f'(s, \omega) f(s, \omega)\, ds\right\} < \infty \tag{1.2}$$

Here M is the symbol for integration with respect to measure P (the mean value) and the prime denotes transposition.

As the loss functional we consider

$$J(u) = \frac{1}{2} M \int_0^T [x_u'(s, \omega) C(s) x_u(s, \omega) + u'(s, \omega) D(s) u(s, \omega)]\, ds \tag{1.3}$$

where $x_u(s, \omega)$ is the solution of system (1.1), corresponding to control $u(s, \omega)$. It is assumed that $C(s)$ is a symmetric nonnegative definite $n \times n$ matrix and $D(s)$ is a symmetric positive definite $m \times m$ matrix. The elements of matrices $C(s)$, $D(s)$ and $D^{-1}(s)$ are nonrandom measurable functions bounded for $s \in [0, T]$.

We introduce the concept of the class U of admissible controls. By F_t we denote the minimal σ-subalgebra of σ-algebra F, relative to which the random vectors $\xi(\omega)$ and $f(s, \omega)$, $s \leqslant t$, are aggregate-measurable. The set $\{F_t\}$ does not monotonically decrease with respect to parameter t, i.e., it forms a stream of σ-algebras. Let $\{E_t\}$ be an arbitrary substream of this stream ($E_t \subset F_t$, $t \in [0, T]$) and $\mathrm{B}E$ be the σ-algebra of subsets of the product $[0, T] \times \Omega$, the subsets being progressively measurable relative to flow $\{E_t\}$. Further, let L_2 $(\mathrm{B}E)$ be the Hilbert space of $\mathrm{B}E$-measurable functions with the scalar product

$$(\varphi, \psi) = M \int_0^T \varphi'(s, \omega) D(s) \psi(s, \omega)\, ds \tag{1.4}$$

Then every element of space $L_2(\mathrm{B}E)$ is called an admissible control, i.e, we set $U \equiv L_2(\mathrm{B}E)$.

Let us explain the physical sense of the introduced definition: here the σ-algebra E_t is interpreted as a collection of the random events connected with the control system, which can be observed up to the instant t; the requirement of $\mathrm{B}E$-measurability of control $u(t, \omega)$ signifies that the magnitude $u_t(\omega)$ of the admissible control at each instant t is chosen with due regard to the information on the behavior of the system's random parameters, contained in σ-algebra E_t.

In practice the collection of observable events can be specified, say, by indicating the set of observable random parameters of the system.

$$E_t = \sigma\,[\xi_i(\omega),\ i = i_1, \ldots, i_l;\ f_k(s, \omega),\ k = k_1, \ldots, k_j,\ s \in S(t) \subset [0, t]]$$

In this case (see [1]) the E_t-measurable random quantity $u_t(\omega)$ admits the representation

$$u_t(\omega) = g_t(\xi_{i_1}(\omega), \ldots, \xi_{i_l}(\omega); f_{k_1}(\cdot, \omega), \ldots, f_{k_j}(\cdot, \omega))$$

where $g_t(\cdot)$ is some measurable (nonrandom) mapping of the space of "trajectories" $\{y_1, \ldots, y_l; z_1(\cdot), \ldots, z_j(\cdot)\}$ into the space R^m, visually demonstrating the explicit dependence of the control function on the observation results. Note that by choosing the stream $\{E_t\}$ we can specify the most diverse classes of control functions. Thus, in the case of $E_t \equiv \{\varnothing, \Omega\}$ we obtain the class of deterministic controls (program control); in that of $E_t \equiv F_{t-\delta}$, $\delta > 0$, we obtain control under lagging information, etc.

The present paper is devoted to the problem of minimizing functional (1.3) on the class U of admissible controls introduced above. Closely related problems were studied earlier in a number of papers (see [2], for instance) on the additional assumption of process f having Markovian property.

2. Existence of the optimal control. Following [3], a measurable random vector-valued function $x_u(t, \omega)$ which almost surely satisfies for each $t \in [0, T]$ a system of integral equations corresponding to (1.1) is called a solution of system (1.1).

L e m m a 1 [3]. Under conditions of Sect. 1 system (1.1) with any admissible control has a unique (to within modification) $\mathrm{B}F$-measurable solution

$$x_u(t,\omega) = \Phi_0(t)\left\{\xi(\omega) + \int_0^t \Phi_0^{-1}(s)\left[B(s)u(s,\omega) + f(s,\omega)\right]ds\right\}. \qquad (2.1)$$

where $\Phi_0(t)$ is the fundamental matrix of solutions of the homogeneous system

$$x^{\cdot} = A(t)x.$$

Lemma 2. Under the conditions of Sect. 1 the optimal element u°: $J(u^\circ) \leqslant J(u)$, $u \in U$, exists in the class of admissible controls.

Proof. By (1.2) we have: $J(0) = c < \infty$. Consider the minimizing sequence $\{u_k\} \subset U$: $u_1 \equiv 0$, $J(u_k) \downarrow \inf J(u)$, $u \in U$. From (1.3) and (1.4) we have

$$\| u_k \|^2 = (u_k, u_k) \leqslant J(u_k) \leqslant c, \quad k \geqslant 1$$

Since a sphere of space $L_2(BE)$ is weakly compact, from sequence $\{u_k\}$ we can separate a weakly convergent subsequence: $u_l \to u^\circ \in L_2(BE)$. Note that since $L_2(BE) \subset L_2(BF)$, the sequence $\{u_l\}$ is weakly convergent also in space $L_2(BF)$. Therefore, from formula (2.1) follows the weak convergence in $L_2(BF)$ of the sequence x_{u_l} to x_{u°. Since the integrand of functional (1.3) is a (downward) convex function of its arguments, from the results in [4] it follows that functional J is lower-semicontinuous relative to sequence $\{u_l\}$

$$\inf_{u \in U} J(u) = \lim_{l\to\infty} J(u_l) \geqslant \lim_{l\to\infty} \inf J(u_l) \geqslant J(u^\circ)$$

But on the other hand, since $u^\circ \in U$, then $J(u^\circ) \geqslant \inf_{u\in U} J(u)$.

3. Optimality criteria. Lemma 3. For an admissible control u° to be optimal, it is necessary and sufficient that it satisfies the following integral minimum principle:

$$\min_{u\in U} H(u) = H(u^\circ) \qquad (3.1)$$

$$H(u) \equiv (\varphi_0, u), \quad \varphi_0(t,\omega) \equiv D^{-1}(t)B'(t)\theta_0(t,\omega) + u^{\circ\cdot}(t,\omega)$$

$$\theta_0(t,\omega) \equiv M\left\{(\Phi_0'(t))^{-1}\int_t^T \Phi_0'(s)\,C(s)\,x^\circ(s,\omega)\,ds \mid E_t\right\}, \quad x^\circ \equiv x_{u^\circ}$$

($M\{\eta \mid E_t\}$ is the conditional mean value of random variable η relative to σ-algebra E_t).

Proof. The necessity of (3.1) follows from the results in [5, 6]. To prove the sufficiency of this condition, we assume that $u^\circ \in U$ satisfies (3.1) and that u is an arbitrary admissible control. We have

$$2J(u) = M\int_0^T \left[(x_u - x^\circ)'C(x_u - x^\circ) + (u-u^\circ)'D(u-u^\circ)\right]ds + \qquad (3.2)$$

$$2M\int_0^T \left[(x^\circ)'Cx_u + (u^\circ)'Du\right]ds - 2J(u^\circ) \equiv J_1 + J_2 - 2J(u^\circ)$$

By virtue of the properties of matrices C and D we have $J_1 \geqslant 0$, where the equality $J_1 = 0$ is achieved only under the condition that $u(t, \omega) = u^\circ(t, \omega)$ for almost all t and ω. We transform J_2 by using (2.1) and (3.1). Changing the order of integration and allowing for the properties of the conditional mean value, we have

$$J_2 = 2M \int_0^T (x^\circ)' C \Phi_0 \left[\xi + \int_0^s \Phi_0^{-1} (Bu + f)\, d\tau \right] ds + 2(u^\circ, u) =$$

$$2 \left\{ M \int_0^T (x^\circ)' C \Phi_0 \left(\xi + \int_0^s \Phi_0^{-1} f\, d\tau \right) ds \right\} + 2M \int_0^T \left(\int_s^T (x^\circ)' C \Phi_0\, d\tau \right) \times$$

$$\Phi_0^{-1} Bu\, ds + 2(u^\circ, u) \equiv 2\{\alpha_1\} + 2(\varphi_0, u) = 2\alpha_1 + 2H(u)$$

Analogously we obtain

$$2J(u^\circ) = \alpha_1 + H(u^\circ) \tag{3.3}$$

Therefore, by virtue of (3.1) and (3.3), from (3.2) follows

$$2J(u) \geqslant \alpha_1 + H(u^\circ) + 2(H(u) - H(u^\circ)) \geqslant 2J(u^\circ)$$

which signifies the optimality of control u°. Note that by virtue of the remark made above about the quantity J_1, the assertion on the uniqueness of the optimal control is also derived from this.

L e m m a 4. For an admissible control u° to be optimal, it is necessary and sufficient that the equality

$$\varphi_0(t, \omega) = 0 \tag{3.4}$$

be satisfied for almost all t and ω.

P r o o f. The sufficiency of condition (3.4) follows immediately from the preceding lemma. Now let $u^\circ \in U$ be the optimal admissible control. Let us assume that (3.4) is violated on a set of positive measure, i.e., $\| \varphi_0 \| > 0$. We set $u_1 \equiv \gamma \varphi_0$, where γ is a positive number. Using the admissibility of u°, it is easy to verify the admissibility of control u_1. But then

$$H(u_1) = \gamma \| \varphi_0 \|^2 < H(u^\circ)$$

when $\gamma < H(u^\circ) \| \varphi_0 \|^{-2}$, which, because of (3.1), contradicts the optimality of control u°.

We combine the results of Lemmas 1–4 in the following theorem.

T h e o r e m. The optimal control problem posed in Sect. 1 has a unique solution. The optimal control function (and only it) satisfies relation (3.4) for almost all t and ω.

4. A u x i l i a r y r e s u l t s. The following propositions will be henceforth repeatedly used.

L e m m a 5. Let the numerical functions $\eta_1(t, \omega)$ and $\eta_2(t, \tau, \omega)$; $t, \tau \in [0, T]$, $\omega \in \Omega$, be measurable with respect to all variables relative to the σ-algebras $\mathrm{B} \times F$ and $\mathrm{B} \times \mathrm{B} \times F$ (B is the σ-algebra of Borel subsets of interval $[0, T]$) and integrable with respect to (t, ω) and (t, τ, ω), respectively. Further, let $\{C_t\}$ be an arbitrary flow of σ-subalgebras, complete with respect to measure P, of σ-algebra F. Then, a selection of variants of conditional mean values exists such that a) the function $\eta_3(t, \omega) \equiv M\{\eta_1(t, \omega) \mid C_t\}$ is progressively measurable relative to flow $\{C_t\}$ and integrable with respect to the variables (t, ω); and b) that

$$M\left\{\int_0^T \eta_2(t, \tau, \omega)\, d\tau \mid C_t\right\} = \int_0^T M\{\eta_2(t, \tau, \omega) \mid C_t\}\, d\tau$$

for almost all t and ω

The lemma's proof follows directly from similar results in [1, 7].

Lemma 6. Let $Y(t, \omega)$ be a solution (in the sense of Sect. 2) of the system of integral equations

$$Y(t) = \alpha(t, \omega) + \int_0^t P(s) Y(s)\, ds$$

where $P(s)$ is an $n \times n$ -matrix with measurable components uniformly bounded on $[0, T]$ and $\alpha(t, \omega)$ is a measurable random vector-valued function with integrable paths. Then

$$Y(t, \omega) = \alpha(t, \omega) + \Psi(t) \int_0^t \Psi^{-1}(s) P(s) \alpha(s, \omega)\, ds$$

almost certainly for each $t \in [0, T]$. Here $\Psi(t)$ is the fundamental matrix of the system $x^{\cdot} = P(t)x$.

The proof follows from the Cauchy formula if we take into account that $Y = K + \alpha$, where K is a solution of the system $K^{\cdot} = PK + P\alpha$, $K(0) = 0$.

5. **Construction of the optimal control.** We seek the optimal control in the form

$$u^\circ(t, \omega) = -D^{-1}(t) B'(t)[G(t) M\{x^\circ(t, \omega) \mid E_t\} + h(t, \omega)] \tag{5.1}$$

Here $x^\circ(t, \omega)$ is, as before, the solution of system (1.1), corresponding to control $u^\circ(t, \omega)$, while $G(t)$ is an $n \times n$ -matrix with deterministic measurable components uniformly bounded on $[0, T]$ and $h(t, \omega)$ is a BE-measurable integrable vector-valued function, both unknown as yet. Substituing (5.1) into formula (2.1), we obtain

$$z^\circ(t, \omega) = \alpha(t, \omega) + \int_0^t P(s) z^\circ(s, \omega)\, ds$$

$$z^\circ(t, \omega) \equiv \Phi_0^{-1}(t) M\{x^\circ(t, \omega) \mid E_t\}, \quad P(t) \equiv -\Phi_0^{-1}(t) \Gamma(t) G(t) \Phi_0(t)$$

$$\Gamma(t) \equiv B(t) D^{-1}(t) B'(t), \quad \alpha(t, \omega) \equiv M\left\{\xi(\omega) - \int_0^t \Phi_0^{-1}[\Gamma h - f]\, ds \mid E_t\right\}$$

From this applying Lemma 6, we have

$$z^\circ(t,\omega)=\alpha(t,\omega)+\Psi(t)\int_0^t \Psi^{-1}(s)P(s)\alpha(s,\omega)\,ds$$

Using this formula, we can establish that when $s > t$

$$M\{z^\circ(s,\omega)\mid E_t\}=\Psi(s)\Psi^{-1}(t)z^\circ(t,\omega)+M\{\beta(t,s,\omega)\mid E_t\} \tag{5.2}$$

$$\beta(t,s,\omega)\equiv\alpha(s,\omega)-\Psi(s)\Psi^{-1}(t)\alpha(t,\omega)+\Psi(s)\int_t^s \Psi^{-1}P\alpha\,d\tau$$

Since u° is the optimal control, by the theorem it satisfies relation (3.4). Using (5.2), from (3.4) we obtain

$$u^\circ(t,\omega)=-D^{-1}(t)B'(t)[I_1(t)z^\circ(t,\omega)+I_2(t,\omega)] \tag{5.3}$$

$$I_1(t)\equiv(\Phi_0'(t))^{-1}\int_t^T \Phi_0'(s)C(s)\Phi_0(s)\Psi(s)\,ds\,\Psi^{-1}(t)$$

$$I_2(t,\omega)\equiv(\Phi_0'(t))^{-1}\int_t^T \Phi_0'(s)C(s)\Phi_0(s)M\{\beta(t,s,\omega)\mid E_t\}\,ds$$

Comparing (5.1) with (5.3), we require the following equalities

$$I_1(t)=G(t)\Phi_0(t),\quad I_2(t,\omega)=h(t,\omega) \tag{5.4}$$

to be satisfied almost certainly for each t. Differentiating the first of these equalities with respect to t, we obtain for the determination of $G(t)$ the known Riccati matrix equation

$$G^\cdot+(GA+A'G)-G\Gamma G+C=0,\quad G(T)=0 \tag{5.5}$$

Under the conditions in Sect. 1, a symmetric nonnegative definite matrix serves as its solution (see [1, 2], for example). Henceforth let $G(t)$ be everywhere the solution of system (5.5). Using (5.2) and Lemma 5, we transform the second equation in (5.4) to the equivalent

$$M\left\{\int_t^T \Phi_0'(s)G(s)[\Gamma(s)h(s,\omega)-f(s,\omega)]\,ds+\Phi_0'(t)h(t,\omega)\mid E_t\right\}=0 \tag{5.6}$$

Let the measurable random vector-valued function $h_1(t,\omega)$ make the expression in (5.6) under the sign of the conditional mean value vanish (almost certainly for each t). Then the BE-measurable vector-valued function $h(t,\omega)\equiv M\{h_1(t,\omega)\mid E_t\}$ (see Lemma 5) satisfies system (5.6). This can be proved by direct substitution into (5.6) taking into account that when $s > t$

$$M\{h(s,\omega)\mid E_t\}=M\{h_1(s,\omega)\mid E_t\}$$

almost certainly. To determine function $h_1(t,\omega)$ we obtain the differential equation system

$$h_1^\cdot+(A'-G\Gamma)h_1+Gf=0,\quad h_1(T,\omega)=0$$

integrating which we find

$$h(t,\omega) = M\left\{\Psi_0(t)\int_t^T \Psi_0^{-1}(s)G(s)f(s,\omega)\,ds \mid E_t\right\} \tag{5.7}$$

Here $\Psi_0(t)$ is the fundamental matrix of system $x^{\cdot} = (G\Gamma - A')x$.

Repeating the preceding arguments, we can now establish that an admissible control satisfying relation (5.1), where $G(t)$ and $h(t,\omega)$ are the solutions of the first and second systems in (5.4), respectively, satisfies relation (3.4) as well, i.e., it is optimal. Let us show that such a control indeed exists and let us find it in explicit form. From (5.1) and (2.1) we have the equation for u°

$$u^\circ(t,\omega) = -D^{-1}(t)B'(t)\left[G(t)\Phi_0(t)\int_0^t \Phi_0^{-1}(s)B(s)u^\circ(s,\omega)\,ds + \gamma(t,\omega)\right] \tag{5.8}$$

$$\gamma(t,\omega) \equiv M\left\{G(t)\Phi_0(t)\left[\xi(\omega) + \int_0^t \Phi_0^{-1}(s)f(s,\omega)\,ds\right] \mid E_t\right\} + h(t,\omega)$$

Assume that $\delta(t,\omega)$ is a solution of the system

$$\delta(t,\omega) = -\Gamma(t)[G(t)v(t,\omega) + \gamma(t,\omega)] \tag{5.9}$$

$$v(t,\omega) = \Phi_0(t)\int_0^t \Phi_0^{-1}(s)\delta(s,\omega)\,ds$$

Then the function

$$u^\circ(t,\omega) \equiv -D^{-1}(t)B'(t)[G(t)v(t,\omega) + \gamma(t,\omega)]$$

satisfies system (5.8), as can be verified by direct substitution. We reduce system (5.9) to the equivalent

$$v^{\cdot} - (A - \Gamma G)v + \Gamma\gamma = 0, \quad v(0,\omega) = 0$$

integrating which we determine the final expression for the optimal control

$$u^\circ(t,\omega) = D^{-1}(t)B'(t)\left[G(t)(\Psi_0'(t))^{-1}\int_0^t \Psi_0'(s)\Gamma(s)\gamma(s,\omega)\,ds - \gamma(t,\omega)\right] \tag{5.10}$$

6. **Examples.** Let us consider certain concrete ways of specifying the flow $\{E_t\}$ of observable events.

Program control. Let $E_t \equiv \{\varnothing, \Omega\}$. In this case every admissible control is, with probability one, independent of the event. The control is effected on the basis of *a priori* information on the regulation system. From (5.7), (5.8) and (5.9) we have

$$\gamma(t,\omega) = \gamma(t) = G(t)\Phi_0(t)\left[M\xi + \int_0^t \Phi_0^{-1}(s)Mf(s)\,ds\right] +$$

$$\Psi_0(t) \int_t^T \Psi_0^{-1}(s) G(s) Mf(s)\, ds$$

$$u^\circ(t, \omega) = u^\circ(t) = D^{-1}(t) B'(t) \left[G(t) (\Psi_0'(t))^{-1} \int_0^t \Psi_0'(s) \Gamma(s) \gamma(s)\, ds - \gamma(t) \right]$$

almost certainly.

Markov case. Let the initial data be nonrandom: $\xi(\omega) \equiv \xi_0$, but the vector-valued function $f(t, \omega)$ be a Markov process satisfying the system of Itô's stochastic differential equations

$$df = l(t, f)\, dt + m(t, f)\, dw, \quad f(0) = f_0 \tag{6.1}$$

where $w(t, \omega)$ is an n-dimensional Wiener process on (Ω, F, P), l be an n-dimensional vector, m be an $n \times n$-matrix. The functions $l_i(t, x)$ and $m_{ij}(t, x)$, $t \in [0, T]$, $x \in R^n$, are assumed to be such that the solution of system (6.1) possesses the transition probability density $p(t, x; s, y)$ satisfying Kolmogorov's inverse equation (see [3]). Let us consider the control method when the whole path of process f is observed: $E_t \equiv \sigma[f(s, \omega), s \leqslant t]$. We have

$$M\{f(s, \omega) \mid E_t\} = M\{f(s, \omega) \mid f(t, \omega)\} = \tag{6.2}$$
$$\int_{R^n} xp(t, f(t, \omega); s, x)\, dx \equiv a(s; t, f(t, \omega))$$

The function $a(s; t, y)$ satisfies the equation (see [3])

$$a(s; t, y) = y + \int_t^s l'(\tau, y) \frac{\partial a}{\partial y}(s; \tau, y)\, d\tau + \tag{6.3}$$
$$\frac{1}{2} \operatorname{Tr} \int_t^s m(\tau, y) m'(\tau, y) \frac{\partial^2 a}{\partial y^2}(s; \tau, y)\, d\tau$$

(Tr stands for the trace of a matrix). We introduce the function

$$N(t, y) \equiv \Psi_0(t) \int_t^T \Psi_0^{-1}(s) G(s) a(s; t, y)\, ds$$

for which, according to (6.2) and (5.7), we have

$$N(t, f(t, \omega)) = h(t, \omega) \tag{6.4}$$

Using (6.3), we can show that this function satisfies the equation

$$\frac{\partial N}{\partial t} + (A' - G\Gamma) N + l'(t, y) \frac{\partial N}{\partial y} + \frac{1}{2} \operatorname{Tr} m(t, y) m'(t, y) \frac{\partial^2 N}{\partial y^2} = 0, \quad N(T, y) = 0$$

Taking into account that the solution x° in the example being examined is consistent with flow $\{E_t\}$, from (5.1) and (6.4) we obtain the following convenient representation for the optimal control

$$u^\circ(t, \omega) = -D^{-1}(t) B'(t) [G(t) x^\circ(t, \omega) + N(t, f(t, \omega))]$$

We note that this result was known earlier (see [2]).

Another representation for the optimal control can be obtained from (5.10), if we take into account that in this case

$$\gamma(t, \omega) = G(t)\,\Phi_0(t)\left\{\xi_0 + \int_0^t \Phi_0^{-1}(s)\, f(s, \omega)\, ds\right\} + N(t, f(t, \omega))$$

The author thanks V. I. Plotnikov for discussions on the work.

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Translated by N. H. C.